# Interactions between Representation Theory, Quantum Field Theory, Category Theory and Quantum Information Theory

***Abstracts***

## Representing Physical Systems as Chu Spaces

**Samson Abramsky**
**Oxford University Computing Laboratory**

We pursue a model-oriented rather than axiomatic approach to the foundations of Quantum Mechanics, with the idea that new models can often suggest new axioms. This approach has often been fruitful in Logic and Theoretical Computer Science. Rather than seeking to construct a simplified toy model, we aim for a `big toy model', in which both quantum and classical systems can be faithfully represented --- as well as, possibly, more exotic kinds of systems.

To this end, we show how Chu spaces can be used to represent physical systems of various kinds. In particular, we show how quantum systems can be represented as Chu spaces over the unit interval in such a way that the Chu morphisms correspond exactly to the physically meaningful symmetries of the systems --- the unitaries and antiunitaries. In this way we obtain a full and faithful functor from the groupoid of Hilbert spaces and their symmetries to Chu spaces. We also consider whether it is possible to use a finite value set rather than the unit interval; we show that two values do not suffice, but three do. We also show a connection between Chu spaces and coalgebras, and make some comparisons between the two frameworks.

## A Possible Approach to Inclusion of Space and Time in Frame Fields of Quantum Representations of Real and Complex Numbers

**Paul Benioff**
**Argonne National Laboratory**

Earlier work on frame fields based on quantum representations of real and complex numbers is expanded to include space and time lattices in each frame in the field. This enables the inclusion of representations of physical systems in each frame. Special emphasis is placed on qukit strings as both mathematical and physical systems (hybrid systems). As physical systems they have a dynamics describable in each frame. As mathematical systems they represent rational numbers. The contents of each frame as seen from a parent frame is considered in some detail. One interesting result is that each point of a $D$ dimensional lattice is a $D$ tuple of hybrid systems in a parent frame with the point location given by the rational number states of the $D$ tuple. It follows that the point locations of the parent frame view of a lattice have energies that are given by the Hamiltonian of the $D$ tuple of hybrid systems.

## Adaptive Coherence Conditioning

**Robert J. Bonneau**
**Air Force Office of Scientific Research**

Many applications in sensing and pattern recognition involve situations where the size of the vector space required to detect and identify a target is unknown a-priori. Examples of this situation occur in sensing applications where the noise subspace is unknown or in pattern recognition where the sample support of the space is not known to distinguish one object from another. We propose a method whereby the measurement vector space can be adaptively resized depending the on the Bayesian risk of the estimators involved.

We will first develop a Lagrange minimization strategy which parallels some recent Lasso techniques of trading coherence for Bayesian risk in selecting the size of the vector space necessary for a given detection problem. We then develop the adaptive coherence updating strategy based on a Newton gradient approach to assess the Bayesian risk at each stage of a detection problem and then update the coherence based on the steepest gradient decent until the permitted risk level is met. We will then show risk bounds for statistical distributions as a function of coherence, show the correspondence in performance to different statistical estimators, and show the impact of the technique in performance of detecting targets in simulated tomographic imagery.

## Quantum Computational Curvature and Jacobi Fields

**Howard E. Brandt**
**U.S. Army Research Laboratory**

Recent developments in the differential geometry of quantum computation are exposited. In the Riemannian geometry of quantum computation [1]-[4], the quantum evolution is described in terms of the special unitary group of n-qubit unitary operators with unit determinant. The group manifold is taken to be Riemannian. To elaborate on several aspects of the methodology, the Riemannian curvature, Jacobi equation, and lifted Jacobi equation on the group manifold are reviewed. Conjugate points and the so-called geodesic derivative are also addressed. This is important for investigations of the global characteristics of geodesic paths in the group manifold, and the determination of optimal quantum circuits for carrying out a quantum computation. Several significant issues of the methodology are emphasized.

[1] M. R. Dowling and M. A. Nielsen, "The Geometry of Quantum Computation," *Quantum Information and Computation* **8**, 0861-0899 (2008).
[2] H. E. Brandt, "Riemannian Geometry of Quantum Computation," to appear in *Nonlinear Analysis* (2008).
[3] H. E. Brandt, "Riemannian Curvature in the Differential Geometry of Quantum Computation," to appear in *Physica E* (2009).
[4] H. E. Brandt, "Riemannian Geometry of Quantum Computation," AMS Short Course Lecture, to appear in Proc. Symposia in Applied Mathematics., American Mathematical Society (2009).

## Perturbative expansions based on the Schrieffer-Wolff transformation

**Sergey Bravyi**
**IBM**

It is widely believed that perturbative series for quantum field theory and many-body physics do not converge but are to be viewed as asymptotic series, meaning that their lowest-order terms provide a good approximation to the quantity of interest while inclusion of higher-order terms may actually give a worse result. We provide a quantitative version of this conjecture which can be rigorously proved for a large class of quantum spin models in which the unperturbed (free) Hamiltonian describes non-interacting spins (qudits) while the perturbation is a k-local Hamiltonian. Our proof relies on the Schrieffer-Wolff transformation of many-body physics which allows one to derive an effective low-energy Hamiltonian preserving locality features of the original Hamiltonian.

This is a joint work with D. DiVincenzo (IBM) and D. Loss (U. Basel)

## A Unified Treatment for the Universality of Quantum Gates for Various Quantum Computing Gates

**Goong Chen**
**Texas A&M University**

Universal quantum gates of one or two qubits can be designed from various quantum computing devices such as cavity-QED, ion and atom traps, quantum dots, linear optics, SQUIDs, etc. The establishment of universality of such devices depends on their Hamiltonians, and the proofs vary accordingly because such Hamiltonians usually are not conjugate to each other. In this talk, we try to find Lie-algebraic approaches to establish a somehow unified connection between the different Hamiltonians in order to obtain the universality of two bit gates. We hope that, eventually, such a unified treatment could facilitate future quantum circuit design where several devices need to be used in a circuit.

This work is a collaboration with V. Ramakrishna.

## Depicting non-locality

**Bob Coecke**
**Oxford University Computing Laboratory**

The punch-line of this talk is to "depict non-locality", that is, to provide a pictorial presentation of the flow of information which, due to Bell/GHZ-type arguments, cannot be given a stochastic spatio-temporal causal explanation. We will show that in the case of qubits the performed computation is that of an AND-gate on the choices of measurements made by distant observers. This work points at a research project that

classifies the computations which nature performs "outside of space-time". It builds further on results on so-called "phase-groups of observables" in:

[1] with Ross Duncan: "Interacting quantum observables". ICALP'08. arXiv:0906.4725
[2] with Bill Edwards and Rob Spekkens: "The group-theoretic origin of quantum non-locality".
http://web.comlab.ox.ac.uk/publications/publication3026-abstract.html

Within an abstract category-theoretic setting we in particular reproduce the result of:

[3] Janet Anders and Dan Browne: "Computational power of correlations" Phys Rev Lett 102, 050502 (2009). arXiv:0805.1002

where it was shown that GHZ-correlations enable to boost parity computations into universal classical computation. Informal introductions to the abstract category-theoretic/diagrammatic framework that constitutes the backbone of our developments are:

[4] "Kindergarten quantum mechanics". arXiv:quant-ph/0510032
[5] "Introducing categories to the practicing physicist". arXiv:0808.1032

Recent extensive tutorial introductions are:

[6] "Quantum picturalism". Contemporary Physics '09. arXiv:0908.1787
[7] "Categories for the practicing physicist". arXiv:0905.3010

## Free Knots: Parity and Cobordisms

**Denis P. Ilyutko, Vassily O. Manturov**
**Moscow State University**

Both classical and virtual knots arise as formal Gauss diagrams modulo some abstract moves corresponding to Reidemeister moves. If we forget about both over/under crossings structure and writhe numbers, we get simplification of virtual knots and links: free knots and free links. Free knots and links are also called homotopy classes of Gauss words and phrases. Obviously, all the free knots corresponding to classical knots are unknot. However, many virtual knots survive after this simplification. We construct invariants of free knots and present their applications to minimality problems of virtual knots. We also investigate cobordisms of free knots. We define a new strong invariant of free knots which allows to detect free knots not cobordant to the trivial one.

## Topological Quantum Information Theory

**Louis Kauffman**
**University of Illinois, Chicago**

This talk will discuss topological models for quantum computation and will concentrate on the Fibonacci model, basing it on their coupling theory associated with the Jones polynomial. We will discuss quantum algorithms for computing the Jones polynomial based on their coupling formalism and also the AJL algorithm of Aharonov, Jones and Landau that is based directly on representations of the Temperley-Lieb algebra. It is intended that this talk be self-contained both for quantum information and for topology.

## Spectrum of the density matrix

**Vladimir Korepin**
**Stony Brook University**

I consider the ground state of a spin chain. The density matrix of a large block of spins is interesting for its description of entanglement. I show that the spectrum is essentially different for the Heisenberg model and for AKLT. In two dimensions, the spectrum is used to calculate topological entanglement.

## An Intuitive Overview of the Theory of Quantum Knots

**Samuel Lomonaco**
**University of Maryland Baltimore County**
**Coauthor: Louis Kauffman**

We present the theory of quantum knots from an intuitive perspective. A quantum knot system is a physically implementable quantum system whose states, called quantum knots, represent the spatial configuration of a closed knotted piece of rope in 3-space. The dynamic behavior of this quantum system according to Schroedinger's equation represents the movement of a closed knotted piece of rope smoothly moving through 3-space in such a way as to not pass through itself. We give two equivalent definitions, one based on mosaic knots, the other on lattice knots. The talk ends with a list of open questions.

## Adventures in Entanglement

**John M. Myers**
**Harvard University**

Quantification of entanglement for mixed 2-qubit states can make use of Wootters' concurrence, but more complex states, pure or mixed, continue to present a puzzle. Motivated by an interest in quantum sensing, we define a degree of entanglement, starting with bipartite pure states and building up to a definition applicable to any mixed state on any tensor product of finite-dimensional vector spaces. For mixed states the degree of entanglement is defined in terms of a minimum over all possible decompositions of the mixed state into pure states. Using a variational analysis we show a property of minimizing decompositions. Combined with data about the given mixed state, this property determines the degrees of entanglement of a given mixed state. For pure or mixed states symmetric under permutation of particles, we show that no partial trace can increase the degree of entanglement. For selected less-than-maximally-entangled pure states, we quantify the degree of entanglement surviving a partial trace.

## Invariants of Knots and Links Arising from Finite-Dimensional Algebras

**David Radford**
**University of Illinois at Chicago**

Finite-dimensional algebras of a certain type over a field *k* possess structures from which regular isotopy invariants of knots and links can be constructed. We develop a general theory for these algebras, develop a general theory for a class of finite-dimensional algebras over *k* from which regular isotopy invariants of oriented knots and links can be constructed, and we consider relationships between algebras in the two classes.

A very important problem which has proven rather daunting is computation of these invariants. Just how rich the collection of invariants described in the preceding paragraph is remains a mystery, a longstanding unsolved problem.

Probably the most intriguing invariant is the one originating in the work of Hennings which arises from the Drinfel'd double *D*(*H*) of certain finite-dimensional Hopf algebras *H* over *k*. How this invariant is related to the Kuperberg's invariant of 3-manifolds derived from *H* is partially understood. Computing the invariant arising from *D*(*H*)could be very illuminating and useful.

The ideas we discuss are for the most part based on the joint work of the author and Louis Kauffman.

## Classifying Modular Categories

**Eric Rowell**
**Texas A&M University**

I will discuss recent developments in the classification of low rank modular categories. In particular, I will discuss joint work with Zhenghan Wang and Richard Stong classifying unitary modular categories of rank <5 and a symbolic computation approach developed with Seung-Moon Hong extending the classification to rank 5 in the non-self-dual case.

## Entangling Power of Braiding Quantum Gates

**Yong-Shi Wu**
**University of Utah**

Recently braiding quantum gates have attracted much attention. Kauffman and Lomonaco first proposed a universal two-qubit braiding quantum gate. Later it was generalized to the three-qubit case, called the GHZ gate. In this talk we report recent results on defining and calculating the entangling power of these quantum gates, in an attempt to quantify a connection between braiding and entanglement entropy. We also discuss whether the braiding gates would have the maximal entangling power. The computation invokes an application of the Cartan decomposition of Lie algebra su($2^N$). This work is a collaboration with Mr. Aaron Ballard.